# Factoring Polynomials

## Type 1 : Removing the Greatest Common Factor(GCF)

Finding the Greatest Common Factor(GCF) is the process of identifying the numbers and variables that a group of terms have in common – in other words, what do they share ?

Example: Factor $4x^2 + 12x$

**Step 1** : Factor each term completely.

$$2 \bullet 2 \bullet x \bullet x \; + \; 2 \bullet 2 \bullet 3 \bullet x$$

**Step 2** : Find all factors that are common (same) in each term.

$$\underline{\underline{2}} \bullet \underline{\underline{2}} \bullet \underline{\underline{x}} \bullet x \; + \; \underline{\underline{2}} \bullet \underline{\underline{2}} \bullet 3 \bullet \underline{\underline{x}}$$

The common factors are: $2 \bullet 2 \bullet x$

Therefore the GCF is $4x$

**Step 3** : Pull out the GCF and then divide each term by it.

$$4x\left( \frac{4x^2}{4x} + \frac{12x}{4x} \right)$$

GCF

Divide each
Term by GCF

**Step 4** : Perform the division by simplifying each term.

$$Ans: 4x\left(x+3\right)$$

$$Example: 4x^3 + 6x - 2x^2$$

$$\underline{\underline{2}} \bullet 2 \bullet \underline{\underline{x}} \bullet x \bullet x \; + \; \underline{\underline{2}} \bullet 3 \bullet \underline{\underline{x}} \; - \; \underline{\underline{2}} \bullet \underline{\underline{x}} \bullet x$$

$$GCF \rightarrow 2 \bullet x \rightarrow 2x$$

$$2x\left(\frac{4x^3}{2x} + \frac{6x}{2x} - \frac{2x^2}{2x}\right)$$

$$2x\left(2x^2 + 3 - x\right)$$

## Type 2 : Factoring Binomials: Difference of Two Squares

Example : Factor $x^2 - 16$

Check to see if the binomial is a difference. Remember that difference indicates a subtraction operation. Each term in the binomial must be a perfect square.

$x^2$ is a "perfect square" because it equals $x \bullet x$

$16$ is a " perfect square" because it equals $4 \bullet 4$

**Step 1** : Find the square root of each term.

$$\sqrt{x^2} = x \qquad \sqrt{16} = 4$$

Term #1 is the square root of the first term $x^2$ and term #2 is the square root of the second term $16$

**Step 2** : Rewrite your binomial as $(Term\#1)^2 - (Term\#2)^2$

$$x^2 - 16 = (x)^2 - (4)^2$$

**Step 3** : Factor into two binomials – one plus and one minus

$$(\underline{\underline{Term\#1}} + \underline{\underline{Term\#2}})(\underline{\underline{Term\#1}} - \underline{\underline{Term\#2}})$$

$$x^2 - 16 = (x-4)(x+4)$$

Example: $4x^2 - 25y^2$

Check: It is a difference and $\sqrt{4x^2} = 2x$ *and* $\sqrt{25y^2} = 5y$

$$4x^2 - 25y^2$$

$$= \left(2x\right)^2 - \left(5y\right)^2$$

$$= \left(2x - 5y\right)\left(2x + 5y\right)$$

Note : A binomial in the form $x^2 + y^2$ **cannot** be factored because it's a sum not a difference

## Type 3 : Trinomials in the form $x^2 + bx + c$ (Coefficient for $x^2$ is 1)

To factor trinomials in this form we must find 2 factors of c with a sum equal to b.

The trinomial $x^2 + bx + c$ factors to

(x + one factor of c)(x + other factor of c)

Remember the two factors of c must have a sum equal to b

Example: Factor $x^2 + 8x + 15$

Find 2 factors of +15
with a sum of +8

The factors of 15 are

$1 \times 15 = 15$ $\qquad$ $1 + 15 = 16$

$3 \times 5 = 15$ $\qquad$ $3 + 5 = 8$

Therefore $x^2 + 8x + 15$ factors to $(x+3)(x+5)$

Example: $x^2 - 2x - 24$

Find 2 factors of -24
with a sum of -2

The factors of -24 are:

| | |
|---|---|
| -1 x 24 | -1 + 24 = 23 |
| 1 x -24 | 1 + -24 = -23 |
| -2 x 12 | -2 + 12 = 10 |
| 2 x -12 | 2 + -12 = -10 |
| -3 x 8 | -3 + 8 = 5 |
| 3 x -8 | 3 + -8 = -5 |
| -4 x 6 | -4 + 6 = 2 |
| 4 x -6 | 4 + -6 = -2 |

The 2 factors we are looking for are 4 and -6

Therefore $x^2 - 2x - 24$ factors to $(x + 4)(x - 6)$

Note : If you cannot find 2 factors of c with a sum of b then the trinomial cannot be factored or we say it's ***prime***.

Example: $x^2 + 7x - 24$

## Type 4 : Trinomials in the form $ax^2+bx+c$

(Coefficient for $x^2$ is greater than 1)

In trinomials where a > 1 we **cannot** find 2 factors of c with a sum of b. These trinomials are factored by a method known as decomposition.

Example: $2x^2+11x+12$

**Step 1** : Find the product of the a coefficient and c coefficient.

$$2x^2+11x+12$$

$$2\times 12$$

$$24$$

**Step 2** : Find 2 factors of your product with a sum equal to b

In this case we are finding 2 factors of 24 with a sum of 11 .

The factors we are looking for are 3 and 8. Using these factors we are going to rewrite $11x$ to equal $3x+8x$

$$2x^2+11x+12$$

$$2x^2+3x+8x+12$$

**Step 3** : Group the first 2 terms and the last 2 terms and remove a common factor from each group.

$$2x^2+3x+8x+12$$
$$\left(2x^2+3x\right)\left(+8x+12\right)$$
$$x\left(2x+3\right)+4\left(2x+3\right)$$

**Step 4** : After you do this you should have a common binomial factor. Now we can write our 2 factors.

$$\left(2x+3\right)\left(x+4\right)$$

Example: $3x^2-5x-2$

$$-6$$

Multiply 3 and -2 to get -6. Two factors of -6 with a sum of -5 are -6 and +1.

$$3x^2-6x+1x-2$$

$$\left(3x^2-6x\right)\left(+1x-2\right)$$
$$3x\left(x-2\right)+1\left(x-2\right)$$
$$\left(x-2\right)\left(3x+1\right)$$

Group and remove the GCF for each group

## Type 5 : Perfect Squared Trinomials

A perfect squared trinomial written in the form $a^2 + 2ab + b^2$ or $a^2 - 2ab + b^2$ and when factored the two binomial factors are the same.

Take note that

1. The first and last terms are perfect squares.
2. The coefficient of the middle term is twice the square root of the first term multiplied by the squared root of the last term.

When we factor perfect squared trinomials we get

$$a^2 + 2ab + b^2 \quad = \quad (a+b)^2$$

$$a^2 - 2ab + b^2 \quad = \quad (a-b)^2$$

Example $\quad x^2 + 12x + 36$

$$(x)^2 + 2(x)(6) + (6)^2$$

$$(x+6)^2$$

Example $\quad 9m^2 - 12mn + 4n^2$

$$(3m)^2 - 2(3m)(2n) + (2n)^2$$

$$(3m-2n)^2$$

## Type 6 : Trinomials with two Variables

These trinomials are factored the same way as trinomials in the form $x^2 + bx + c$ and $ax^2 + bx + c$ but each binomial factor will have a variable in each term.

Example: $x^2 + 4xy + 3y^2$

Find 2 factors of 3 with a sum of 4

The two factors will be in the form $(x + ?\,y)(x + ?\,y)$ where **?** is replaced with the two factors of 3 with a sum of 4.

$$x^2 + 4xy + 3y^2 \quad = \quad (x + 1y)(x + 3y)$$

Example: $2a^2 - 7ab + 3b^2$

Factors are in the form (? a + ? b)(? a + ? b) because a > 1

$$2 \times 3$$

$$6$$

Find 2 factors of 6 with a sum of -7

$$2a^2 - 1ab - 6ab + 3b^2$$

Group and remove the GCF for each group

$$a(2a - b) - 3b(2a - b)$$

$$(2a - b)(a - 3b)$$