Mathematics 3201
Sample Mid-Year Exam #2, 2014-15
Item Breakdown/Solutions

PART I:

#	Ans	2M	2A	L3	Guide	Outcome
1	A	X			p.22	LR2
2	A		X		p.24	LR2
3	B	X			p.26	LR2
4	C		X		p.26	LR2
5	D		X		22/30	LR2
6	A		X		p.34	LR2
7	D	X			p.30	LR2
8	B	X			p.52	P4
9	D	X			52/58	P4/P5
10	C	X			p.58	P5
11	D		X		p.60	P5
12	B	X			p.64	P5
13	A		X		p.68	P5
14	A		X		p.68	P5
15	C	X			p.70	P5/P6
16	B		X		70/72	P6
17	A		X		p.72	P6
18	C	X			80/82	P1

#	Ans	2M	2A	L3	Guide	Outcome
19	D	X			p.92	P3
20	A		X		p.88	P2
21	D		X		p.88	P2
22	B		X		p.84	P5
23	B		X		p.92	P3
24	C			X	p.84	P6
25	D			X	92/94	P3
26	B	X			p.100	RF1
27	A	X			p.100	RF1
28	C	X			p.102	RF1
29	B		X		p.104	RF1
30	D		X		p.104	RF1
31	A		X		p.106	RF2
32	D		X		p.106	RF2
33	B	X			p.108	RF2
34	C		X		p.108	RF2
35	C			X	p.114	RF3

PART II:

Item	Value	L2A	L3	Guide	Outcome
36a	3	X		p.34	LR2
36b	3	X		p.30	LR2
37a	2	X		p.64	P5
37b	2	X		52/53	P4
37c	3	X		p.74	P5/P6
37d	3		X	p.64	P4
38a	3	X		84/86	P5

Item	Value	L2A	L3	Guide	Outcome
38b	3		X	80/84/92	P1/P3/P6
38c	3		X	p.83	P6
39ai	1		X	p.106	RF2
39aii	3	X		p.106	RF2
39b	2	X		p.106	RF2
39c	4	X		p.114	RF3

PART II - Total Value: 35 marks

Answer **ALL** items in the space provided. Show **ALL** workings.

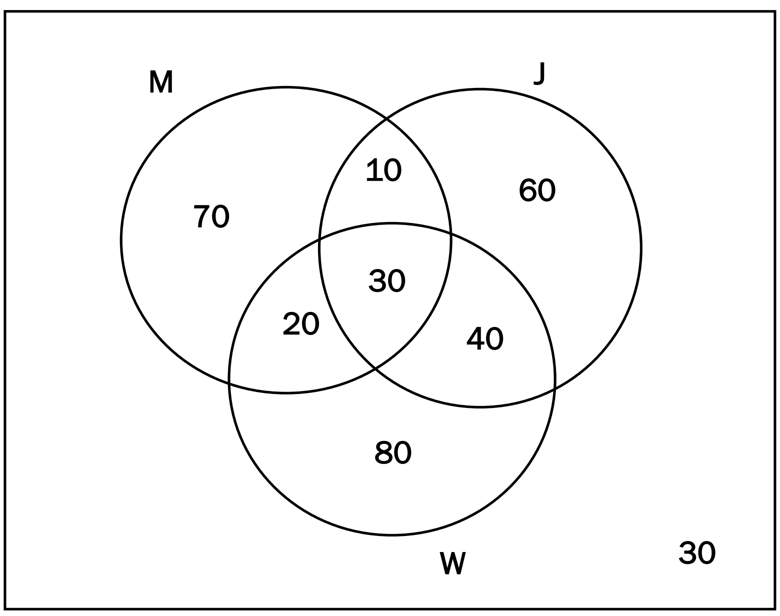
Value

3 36(a). Students were surveyed on what beverage(s) they have ordered at the cafeteria.

- 130 ordered milk
- 140 ordered juice
- 170 ordered water
- 40 had ordered juice and milk
- 70 had ordered juice and water
- 50 had ordered milk and water
- 30 had ordered all three
- 30 had not ordered any beverage

Draw a Venn diagram to illustrate this information and use it to determine how many students were surveyed.

Venn diagram **(2 marks)**



$$x = 30 + 10 + 40 + 20 + 70 + 60 + 80 + 30$$
$$x = 340$$ **(1 mark)**

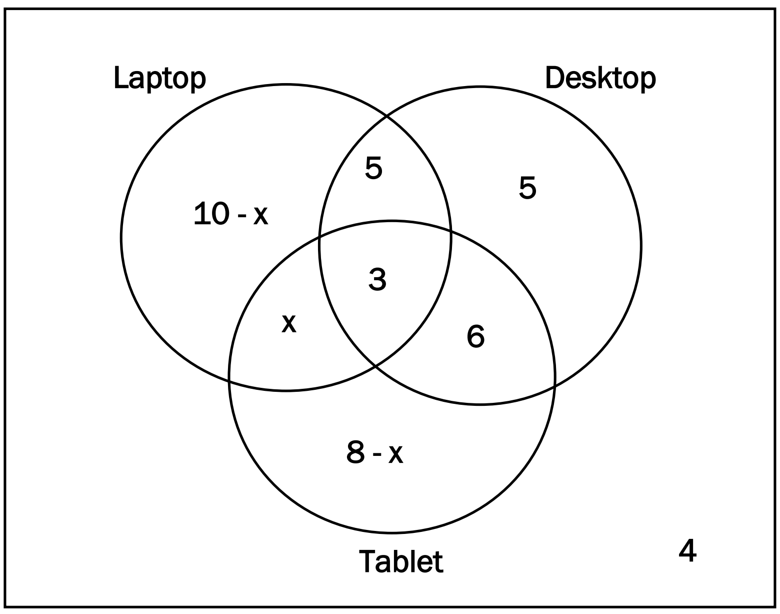
The # of students surveyed was 340.

Value

3 36(b). In a group of 35 people:

- 18 people own a laptop computer
- 19 people own a desktop computer
- 17 people own a tablet computer
- 5 people own a laptop and a desktop but not a tablet
- 6 people own a desktop and a tablet but not a laptop
- 3 people own all three types of computers
- 4 people do not own either type of computer

Determine the number of people who own a laptop and a tablet, but not a desktop.



$$(10 - x) + x + (8 - x) + 3 + 5 + 6 + 5 + 4 = 35$$ (1.5 marks)

$$41 - x = 35$$ (0.5 mark)

$$-x = -6$$ (0.5 mark)

$$x = 6$$ (0.5 mark)

The # of children who own a laptop and a tablet, but not a desktop, is 6.

2 37(a). In how many ways can a teacher arrange 9 students in a line if Alice, Bob, and Carol must be seated together?

$7! \times 3!$ (1.5 marks)

30240 (0.5 mark)

Value

2 37(b). In Newfoundland and Labrador a license plate consists of a letter-letter-letter-digit-digit-digit arrangement such as CRT 123.

i) How many arrangements are possible if a license plate must start with C and end in 3 when repetition is allowed?

$$\mathbf{1 \times 26 \times 26 \times 10 \times 10 \times 1 = 67600}$$ **(1 mark)**

ii) How many arrangements are possible if a license plate must start with C and end in 3 when repetition is not allowed?

$$\mathbf{1 \times 25 \times 24 \times 9 \times 8 \times 1 = 43200}$$ **(1 mark)**

3 37(c). Algebraically solve for n: $_nC_2 = 120$

$$\frac{n!}{2!(n-2)!} = 120$$

$$\frac{n(n-1)(n-2)!}{2!(n-2)!} = 120$$ **(0.5 mark)**

$$\frac{n(n-1)}{2} = 120$$

$$n(n-1) = 240$$ **(0.5 mark)**

$$n^2 - n = 240$$

$$n^2 - n - 240 = 0$$ **(0.5 mark)**

$$(n-16)(n+15) = 0$$ **(0.5 mark)**

$n = 16$ *or* ~~$n = -15$~~ **(1 mark)**

3 37(d). Given the digits 1, 2, 3, 4, and 5, how many two or three digit even numbers can be made if repetition is not allowed?

$(4 \times 2) \rightarrow$ **8 *two digit even numbers no repetition*** **(1 mark)**

$(4 \times 3 \times 2) \rightarrow$ **24 *three digit even numbers no repetition*** **(1.5 marks)**

32 *two or three digit even numbers no repetition* **(0.5 mark)**

Value

3 38(a). Nine horses are entered in a race and each one is equally likely to win. To the nearest percent, determine the probability that one horse, Mr. Mal, will not finish in the top three. Show your workings.

$$P(Mr.Mal\ in\ top\ three) = \frac{8!+8!+8!}{9!} = \frac{3(8!)}{9!} \quad \textbf{(1 mark)}$$

$$P(Mr.Mal\ in\ top\ three) = \frac{3(8!)}{9\times 8!} = \frac{3}{9} \quad \textbf{(0.5 mark)}$$

$$P(Mr.Mal\ not\ in\ top\ three) = 1 - \frac{3}{9} = \frac{6}{9} \quad \textbf{(1 mark)}$$

=67% **(0.5 mark)**

OR Probability of Mr. Mal ending up in either of the 9 places is $\frac{1}{9}$ **(0.5 mark)**. Therefore, if not in the top 3, then for each position 9th through 4th: $\frac{1}{9}+\frac{1}{9}+\frac{1}{9}+\frac{1}{9}+\frac{1}{9}+\frac{1}{9}$ **(1.5 marks)** $=$ $\frac{6}{9}$ **(0.5 mark)**

=67% **(0.5 mark)**

3 38(b). There are 6 blue marbles, 3 red marbles, and 1 green marble in a bag. If you reach in and randomly select 2 marbles from the bag, what are the odds of them both being blue?

$$P(2\ blue\ marbles) = \frac{6}{10}\times\frac{5}{9} = \frac{1}{3} \quad \textbf{(1 mark)}$$

Therefore, $P(not\ 2\ blue\ marbles) = 1 - \frac{1}{3} = \frac{2}{3}$ **(0.5 mark)**

Therefore, odds of them both being blue: $\frac{\frac{1}{3}}{\frac{2}{3}}$ → 1:2 **(1.5 marks)**

OR P(both blue)= $\frac{{}_6C_2}{{}_{10}C_2} = \frac{\frac{6!}{2!4!}}{\frac{10!}{2!8!}} = \frac{\frac{6\times5}{2}}{\frac{10\times9}{2}} = \frac{15}{45} = \frac{1}{3}$ **(1 mark)**

Therefore, $P(not\ 2\ blue\ marbles) = 1 - \frac{1}{3} = \frac{2}{3}$ **(0.5 mark)**

Therefore, odds of them both being blue: $\frac{\frac{1}{3}}{\frac{2}{3}}$ → 1:2 **(1.5 marks)**

3 38(c). A committee of 4 people is chosen at random from 5 married couples. What is the probability that the committee contains no married couples?

There are $({}_{10}C_4)$ *ways to choose 4 committee members from 10 people.*

There are $({}_5C_4)$ *ways to choose 4 couples and there are* $({}_2C_1)$ *ways to choose 1 personfrom each couple.*

Therefore, $P(committee\ contains\ no\ individuals\ married\ to\ each\ other) =$

$\frac{{}_5C_4\times{}_2C_1\times{}_2C_1\times{}_2C_1\times{}_2C_1}{{}_{10}C_4}$ **(2 marks)** $=$

$\frac{5\times2\times2\times2\times2}{\left(\frac{10\times9\times8\times7}{4\times3\times2\times1}\right)}$ **(0.5 *mark*)** $\frac{80}{210}$ **(0.5 *mark*)** $= \frac{8}{21}$

Value

39(a). Karen simplified an expression as follows:

$$\frac{2x+8}{x^2-16}-\frac{x}{2x-8}$$

$$=\frac{x+4}{x^2-16}-\frac{x}{x-4} \qquad \text{Step 1}$$

$$=\frac{x+4}{(x+4)(x-4)}-\frac{x}{x-4} \qquad \text{Step 2}$$

$$=\frac{1}{x-4}-\frac{x}{x-4} \qquad \text{Step 3}$$

$$=\frac{1-x}{x-4} \qquad \text{Step 4}$$

1 (i) Identify the step in which the error occurred and explain the mistake.

The error occurred in Step 1. (0.5 mark) She divided $2x+8$ and $2x-8$ by 2 instead of factoring out a 2. (0.5 mark)

3 (ii) Correct the error and simplify.

$$\frac{2\cancel{(x+4)}}{\cancel{(x+4)}(x-4)}-\frac{x}{2(x-4)} \qquad \textbf{(1.5 marks)}$$

$$=\frac{2}{x-4}-\frac{x}{2(x-4)}$$

$$=\frac{4}{2(x-4)}-\frac{x}{2(x-4)} \qquad \textbf{(0.5 mark)}$$

$$=\frac{4-x}{2(x-4)} \qquad \textbf{(0.5 mark)}$$

$$=-\frac{1}{2} \qquad \textbf{(0.5 mark)}$$

2 39(b). Simplify: $\frac{10(x-3)}{4x+24} \div \frac{x^2-9}{x^2-36}$, $x \neq -6, -3, 3, 6$

$$=\frac{10(x-3)}{4x+24}\times\frac{x^2-36}{x^2-9} \qquad \textbf{(0.5 mark)}$$

$$=\frac{10\cancel{(x-3)}}{4\cancel{(x+6)}}\times\frac{(x-6)\cancel{(x+6)}}{\cancel{(x-3)}(x+3)} \qquad \textbf{(1 mark)}$$

$$=\frac{10(x-6)}{4(x+3)} \qquad \textbf{(0.5 mark)}$$

$$=\frac{5(x-6)}{2(x+3)}$$

Value

4 39(c). A school volleyball team and its chaperones are going to a tournament out of the province that has a total cost of \$7200. The cost of the trip is to be divided amongst everyone going. At the last minute, two people get sick and cannot attend, increasing the cost per person by \$40. If x represents the number of people travelling and the situation is modelled by $\frac{7200}{x-2} - \frac{7200}{x} = 40$, algebraically determine the number of people who originally planned to attend the tournament.

$\mathbf{x\cancel{(x-2)}\left[\frac{7200}{\cancel{x-2}}\right] - \cancel{x}(x-2)\left[\frac{7200}{\cancel{x}}\right] = 40x(x-2)}$ **(0.5 mark)**

$\mathbf{7200x - (7200x - 14\,400) = 40x^2 - 80x}$ **(0.5 mark)**

$\mathbf{\cancel{7200x} - \cancel{7200x} + 14\,400 = 40x^2 - 80x}$ **(0.5 mark)**

$\mathbf{40x^2 - 80x - 14\,400 = 0}$

$\mathbf{x^2 - 2x - 360 = 0}$ **(1 mark)**

$\mathbf{(x-20)(x+18) = 0}$ **(0.5 mark)**

$\mathbf{x = 20; x \neq -18}$

−18 is an extraneous root **(0.5 mark)**

20 people **(0.5 mark)**